
46
DIONYSIS KONSTANTINOU · ANDREAS MEIER · ZBIGNIEW TRZMIEL
DON'T TOUCH
THE GROUND

motion, rotation, rolling motion, translational kinetic energy, rotational kinetic energy, friction

physics, ICT

Two sets of activities are provided. The first is suitable for students 14–15 years old, and both sets are suitable for 16–18 years old.

1 | SUMMARY

Students study the bounce of a ball in terms of motion, kinetic energy and momentum. They also discover that the kinetic energy of a real body consists of both translational and rotational kinetic energy.

2 | CONCEPTUAL INTRODUCTION

2 | 1 Abstract

Goalkeepers say that their job becomes more difficult if the ball bounces on the ground in front of them. In this teaching unit we show students how to investigate the factors that cause changes in the energy and motion of a ball when it bounces. In this context, students will encounter the laws of physics related to the translational and rotational motion of a solid body, especially with respect to a rolling motion. Two experiments are at the core of the unit. Students record the motion of a ball and analyse this with a video-analysis tool. Experiments have been chosen in a way that gives students the opportunity to study the respective phenomenon. As such, they will reach conclusions and will be able to explain the bounce of a ball in terms of force, motion, momentum and energy.

2 | 2 Required knowledge

Students should be familiar with the physics of motion, the role of force in motion, and potential and kinetic energy with respect to point masses. They also should be able to work with vector magnitudes such as velocity and linear momentum.

2 | 3 Theoretical background

2 | 3 | 1 Kinetics

Rolling motion is a combination of translational and rotational motion. In this type of motion:

1. The centre of mass (cm) moves with a translational motion. Its velocity with respect to the ground is $\vec{v}_{cm}$.

2. The rest of the body rotates around the centre of mass and demonstrates two types of motion, i.e. translational motion with $\vec{v}_{cm}$ and rotational motion.

Let us consider point i of the body. In the second type of motion its absolute velocity, with respect to its cm, is $v^i_{rel,cm} = r_i\,\omega$.

Angular velocity is located on the axis of rotation. The velocity of point i with respect to cm is tangential to the path of point i. The two velocities are connected by the right-hand-rule.

r_i: distance of the specific point i from the axis of rotation [m]
ω: angular velocity of the body $[\frac{1}{s}]$
v: velocity $[\frac{m}{s}]$

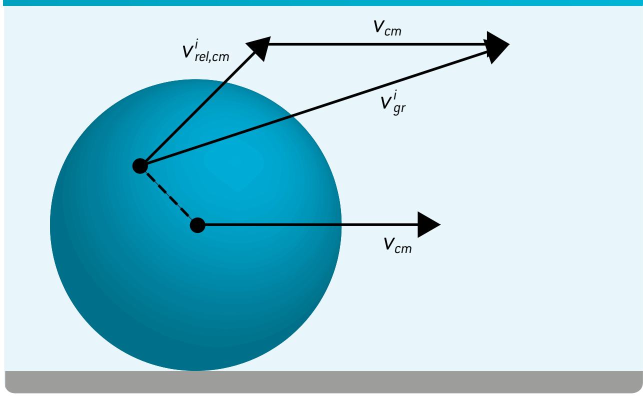
With respect to the points of the circumference, their $\vec{v}_{rel,cm}$ becomes $R\omega$.

R: radius of the body [m]

Therefore, the velocity of point i of the body with respect to the ground is the vector sum of the two velocities **(FIG. 1)**.

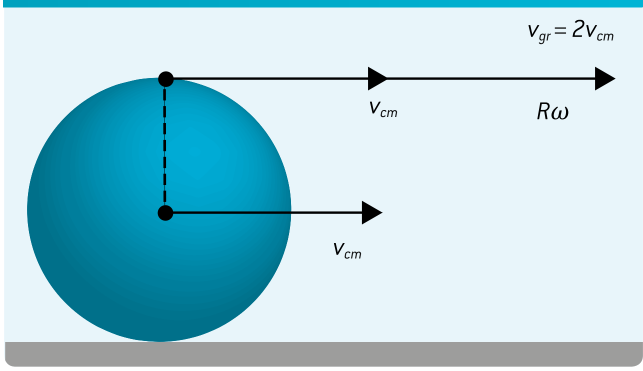
$$\vec{v}^i_{gr} = \vec{v}_{cm} + \vec{v}^i_{rel,cm}$$

FIG. 1



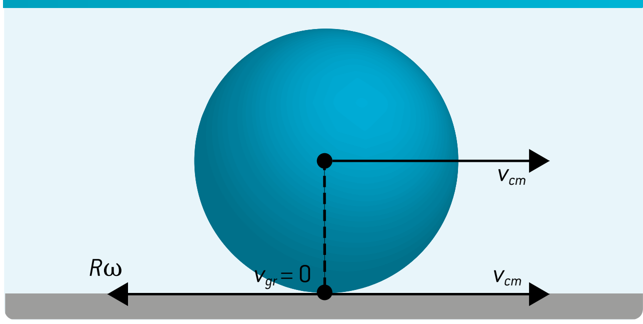
The $\vec{v}_{gr}$ of the uppermost point of the body is equal to $\overrightarrow{2v}_{cm}$.

FIG. 2



The velocity $\vec{v}_{gr}$ of the point in contact with the ground is zero, i.e. it is momentarily at rest **(FIG. 3)**.

FIG. 3



Finally, the condition $v_{cm} = R\omega$ means that the body is rolling without slipping.

2|3|2 Kinetic energy

A moving spherical body has, in general, translational and rotational kinetic energy: $E_{kin,tr}$ and $E_{kin,rot}$ respectively.
$E_{kin,tr} = \frac{1}{2} mv^2$ and $E_{kin,rot} = \frac{1}{2} I\omega^2$

m: mass [kg]
I: moment of inertia [$kg \cdot m^2$]
v: absolute velocity [$\frac{m}{s}$]
ω: angular velocity of the spherical body [$\frac{1}{s}$]

Let us consider such a body as it hits the ground, and let us focus on the short space of time, just before and just after impact, in which we can investigate the force acting between the body and the ground.

Before impact:

$E_{kin,tr(1)} = \frac{1}{2} mv_1^2$ and $E_{kin,rot(1)} = \frac{1}{2} I\omega_1^2$.

After impact these two quantities still exist but with different values:
$E_{kin,tr(2)} = \frac{1}{2} mv_2^2$ and $E_{kin,rot(2)} = \frac{1}{2} I\omega_2^2$.

The indices 1 and 2 correspond to the values before and after impact on the ground.

The force acting between the ground and the body consists of vertical and horizontal components. If we assume that the ball does not slip on the ground, the horizontal component is static friction. Its work on the ball is zero, while its torque causes angular acceleration. This means that the angular velocity changes in magnitude and sometimes in direction. Nevertheless, no energy is converted to heat, and we only get an exchange between translational and rotational energy. The vertical component and the weight of the ball produce vertical acceleration with respect to the ball. Given that the ball does not slide on the ground, we can apply the principle of conservation of mechanical energy:
$E_{pot(1)} + E_{kin,tr(1)} + E_{kin,rot(1)} = E_{pot(2)} + E_{kin,tr(2)} + E_{kin,rot(2)}$.

E_{pot} is potential energy, while the indices 1 and 2 refer to the states just before and just after the ball bounces.

Since we focus on the event of the ball bouncing on the ground,
$E_{pot(1)} = E_{pot(2)}$

and $E_{kin,tr(1)} + E_{kin,rot(1)} = E_{kin,tr(2)} + E_{kin,rot(2)}$.

As a result of several factors, including the surface of the ground and the angular velocity of the ball just before impact, it is difficult to estimate the effect of friction. Therefore, it is not easy to predict the data relating to the motion of the ball just after bouncing, especially the vector of its velocity.

2|4 Experiments and procedures

1. In order to awaken their interest, students are asked to drop a ball while simultaneously imparting initial rotation [1]. Hopefully, students will associate the "kick" of the ball with the spin that has been imparted.

2. First experiment (first set of activities)
   Students assemble a ramp consisting of two parallel bars. The distance between these two bars should be somewhat less than the diameter of the ball.

FIG. 4 The set-up for the first experiment.

Students are asked to release a small ball from the top of the ramp, record its motion and analyse this with a video-analysis tool, for example Tracker [2]. An extended presentation of this software can be found in the publication *iStage 1 – Teaching Materials for ICT in Natural Sciences* [3]. It would be even better to use a "fast" camera (120 frames per second or more).

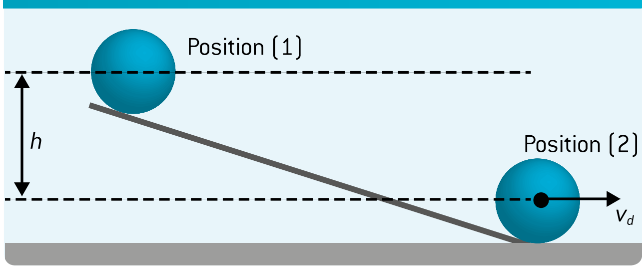
The solid ball (m, R) $I = \frac{2}{5} mR^2$ rolls without slipping from position (1) to the ground, i.e. position (2), and continues rolling along the ground **(FIG. 5)**.

Remark: The moment of inertia for a ball used in football games is closer to $\frac{2}{3} mR^2$.

In the experiments a solid ball is used.

As the ball rolls down the ramp, its velocity v and angular velocity ω are changing according to $v = R\omega$.

FIG. 5



The principle of energy conservation is as follows:
$mgh = \frac{1}{2}mv_d^2 + \frac{1}{2}I\omega^2 = \cdots = \frac{7}{10}mv_d^2$.

$\vec{v}_d$ is the velocity of the ball at the base of the ramp. The translational kinetic energy is equal to $\frac{5}{10}mv_d^2$, and therefore the rotational kinetic energy is equal to $\frac{2}{10}mv_d^2$.

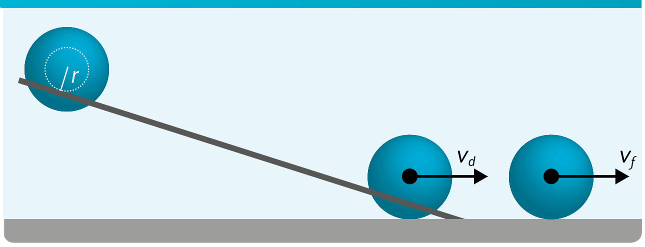
Therefore $\frac{E_{kin,rot}}{E_{kin,tr}} = \frac{2}{5}$.

In the proposed experiment, the motion of the ball on the ramp is according to $v = r\omega$, where r is the distance between the axis of rotation and the points at which the ball touches the ramp.

The experiment is set up **(FIG. 6)** such that $r < R$. Consequently the ratio $\frac{E_{kin,rot}}{E_{kin,tr}}$

is greater than $\frac{2}{5}$. Once the ball is on the ground, this will become equal to $\frac{2}{5}$, so that the rolling motion will assume a new configuration, whereby the distance between the axis of rotation and the point at which the ball touches the ground is equal to R.

FIG. 6



This is exactly what happens and, after a very rapid transition, the velocity of the ball assumes its final value, whereby the velocity $\vec{v}_f$ is greater than the velocity $\vec{v}_d$, with which the ball meets the ground.

Students can see, even with the naked eye, that the ball travels faster on the ground. They can then analyse the motion and define the velocities $\vec{v}_d$ and $\vec{v}_f$.

To do so, they must consider the rotational kinetic energy. Otherwise, there is no explanation in terms of energy conservation. Anyone who is aware of the fact that a solid body can have translational and rotational kinetic energy will understand that some of the rotational kinetic energy has been transformed into translational kinetic energy as a result of the friction between the ground and the ball.

2 | 5 Materials required

Two bars of 1 metre in length and the relevant stands and connectors; one small ball, preferably solid and made of hard rubber. A typical school laboratory is undoubtedly equipped with these materials.

FIG. 7 First part of the motion, v_d = 1.85 m/s

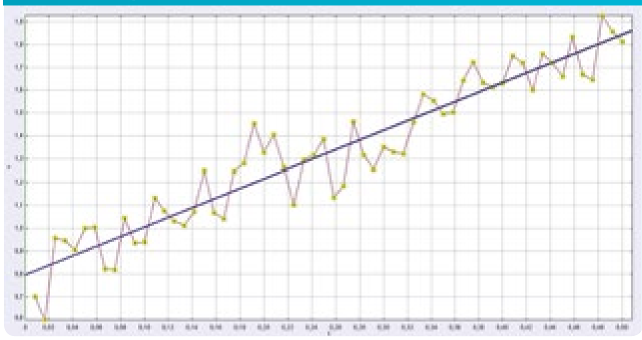
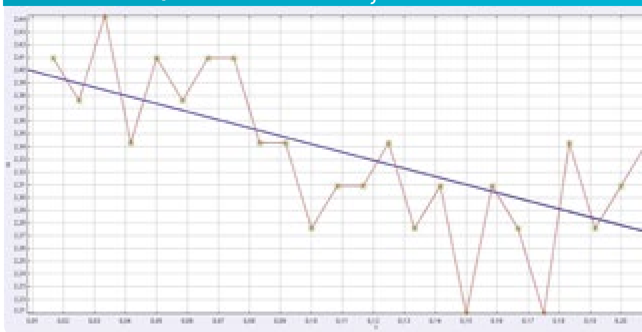


FIG. 8 Second part of the motion, v_f = 2.4 m/s



3 | WHAT THE STUDENTS DO

3 | 1 First experiment: first set of activities

1. Set up the experiment.
2. Record a video [1].
3. Proceed with a video analysis tool, for example Tracker [2].
4. Define the velocities just before and just after impact with the horizontal plane (see **FIGS. 6** and **7**).
5. Measure the radius of the ball and define its angular velocity when it starts rolling along the ground **(FIG. 9)**.
6. Measure the mass of the ball and define the translational kinetic energy just before ($E_{kin,tr(1)}$) and just after ($E_{kin,tr(2)}$) impact with the horizontal plane **(FIG. 9)**.
7. Explain the change in kinetic energy.

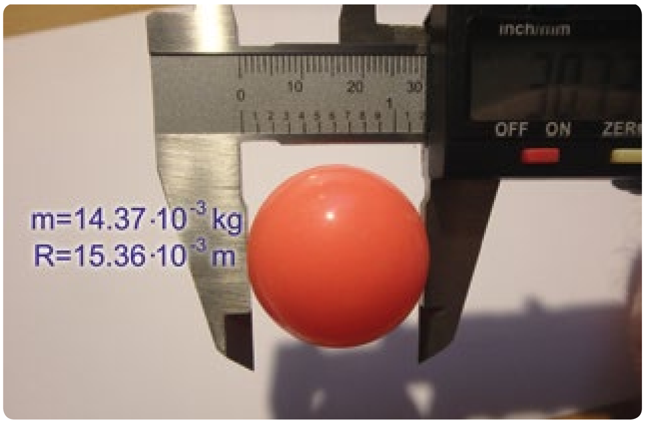


FIG. 9 ω = 156 s^{-1}, $E_{kin,tr(1)} = 2.46 \cdot 10^{-2}$ J, $E_{kin,tr(2)} = 4.14 \cdot 10^{-2}$ J

FIG. 10 The set-up for the second experiment

3 | 2 Second experiment

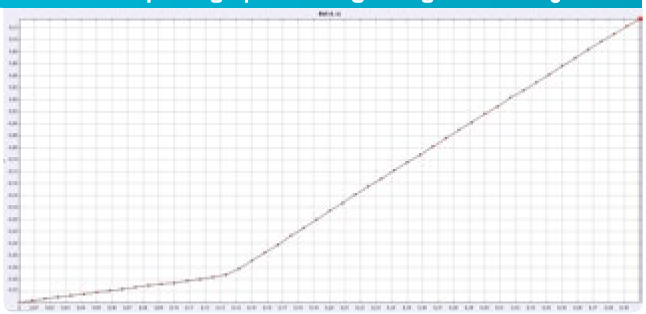
Students should set up an experiment similar to the first one. This time, however, the end of the ramp should be positioned about 0.6 metres above the horizontal plane.

Students should let the ball roll and fall on the surface below. They should record the motion and analyse it with a video-analysis tool, e.g. Tracker [2]. In this case the interesting aspect of the motion begins once the ball has left the ramp, when it assumes a remarkable spin. In this experiment, students will penetrate deeper into the fields of motion and energy.

Second set of activities

1. Set up the experiment
2. Let a ball roll downwards from the top of the ramp and record the motion with a camera [1].
3. Plot a graph of x vs. t and define the horizontal component of the velocity of the ball v_x as it falls and as it rises. Explain the change in v_x.

FIG. 11 Example of graph showing change of velocity



4. Measure the mass of the ball and calculate how much of the ball's $E_{kin,rot}$ is transformed into $E_{kin,tr}$. You should also define the velocity of the ball just before and just after it bounces.

$v_{fall,fin} = 2.55\,\frac{m}{s}$ $E_{kin,tr(1)} = 4.67 \cdot 10^{-2}$J **(FIG. 12)** and

$v_{rise,init} = 2.76\,\frac{m}{s}$ $E_{kin,tr(2)} = 5.47 \cdot 10^{-2}$J **(FIG. 13)**

$\Delta E_{kin,tr} = 0.8 \cdot 10^{-2}\text{J} = -\Delta E_{kin,rot}$

FIG. 12

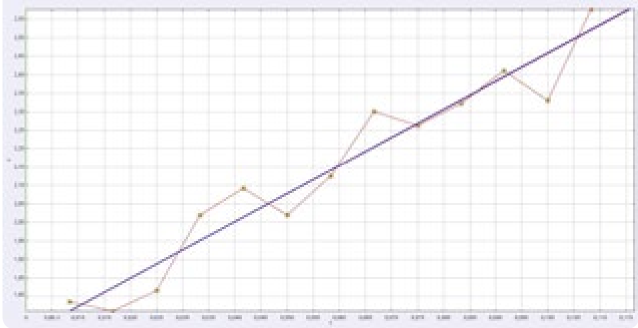
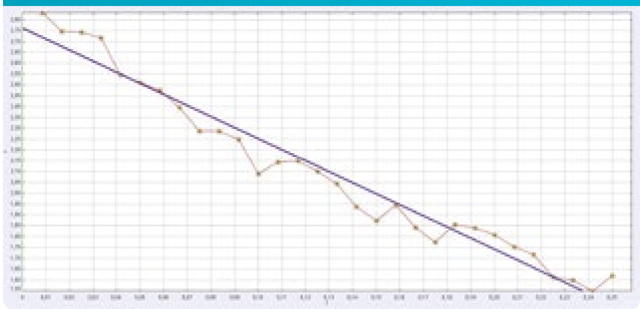


FIG. 13



5. Define the change $\overrightarrow{\Delta p}$ [kg $\cdot \frac{m}{s}$] in momentum of the ball during its contact with the ground.
$\overrightarrow{\Delta p} = m\overrightarrow{\Delta v}$

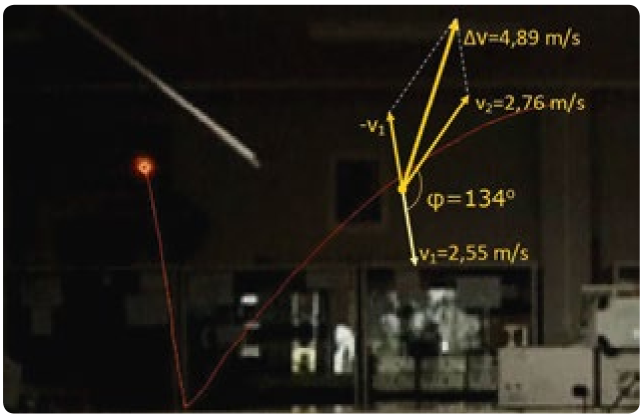


FIG. 14

$\vec{v_1}$ and $\vec{v_2}$ are the velocities just before and just after the bounce. Their absolute values in the specific experiment are $2.55\,\frac{m}{s}$ and $2.76\,\frac{m}{s}$ respectively with an angle of $\varphi = 134°$ between them.

$\overrightarrow{\Delta v}$ is the change in velocity. Its absolute value is calculated to be $4.89\,\frac{m}{s}$. The angle between $\vec{v_2}$ and $\overrightarrow{\Delta v}$ is calculated to be an angle of 24°.

The change in momentum results from the formula
$\overrightarrow{\Delta p} = m\,\overrightarrow{\Delta v}$.

Its direction is the same as the direction of $\overrightarrow{\Delta v}$ and its absolute value is $7 \cdot 10^{-2}$ kg $\cdot \frac{m}{s}$.

6. Consider the second part of the motion as if the ball were thrown from ground level. Define the initial magnitudes that characterise this throw and calculate the maximum height and the range of the throw. Compare the values you have determined with the respective values from Tracker. Explain any differences between the data analysis and the theoretical values.

4 | CONCLUSION

Students should observe the changes in motion and energy of a ball and relate these changes to the force, especially its horizontal component, acting between the ball and the ground, and to the torque of this force. At the same time they should conclude that the kinetic energy of a solid body consists of two quantities (translational and rotational kinetic energy). Finally, they might also overcome some preconceptions that perhaps derive from the fact that we usually work with the point-mass model while teaching mechanics.

5 | COOPERATION OPTIONS

Students of different schools, not necessarily from the same country, can communicate and exchange videos, primarily with respect to the first activity. It is assumed that they will reach the same conclusions, which they can then discuss via a teleconference.

Finally, they can meet up and conduct a set of activities, such as:

1. Go outside and set up a video camera. Record a video of a ball falling on the ground and look at the data for the motion of the ball during its impact with the ground.
2. Analyse this motion.
3. Draw conclusions about the characteristics of the friction during the impact of the ball with the ground.
4. Define the velocity of the ball before and after it hits the ground, measure the mass of the ball and calculate the translational kinetic energy.
5. Ask a skilled player from the class to kick a ball with varying techniques, record videos and describe the results when the ball hits the ground.
6. Produce a definitive answer to the crucial question as to why goalkeepers have greater difficulty when the ball bounces on the ground in front of them.
7. Once other activities have been completed, play a football game devoted to science. Naturally, such a game will produce a win-win situation for both sides, no matter the ultimate score!

RESOURCES

[1] www.science-on-stage.de/iStage3_materials
[2] www.physlets.org/tracker
[3] www.science-on-stage.de/iStage1-download

IMPRINT

TAKEN FROM
iStage 3 - Football in Science Teaching
available in Czech, English, French, German,
Hungarian, Polish, Spanish, Swedish
www.science-on-stage.eu/istage3

PUBLISHED BY
Science on Stage Deutschland e.V.
Poststraße 4/5
10178 Berlin · Germany

REVISION AND TRANSLATION
TransForm Gesellschaft für Sprachen- und Mediendienste mbH
www.transformcologne.de

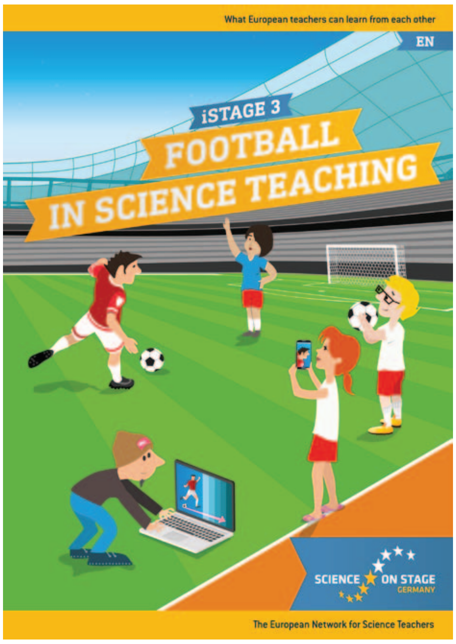
CREDITS
The authors have checked all aspects of copyright for the images and texts used in this publication to the best of their knowledge.

DESIGN
WEBERSUPIRAN.berlin

ILLUSTRATION
Tricom Kommunikation und Verlag GmbH
www.tricom-agentur.de

PLEASE ORDER FROM
www.science-on-stage.de
info@science-on-stage.de

First edition published in 2016



SCIENCE ON STAGE – THE EUROPEAN NETWORK FOR SCIENCE TEACHERS

... is a network of and for science, technology, engineering and mathematics (STEM) teachers of all school levels.
... provides a European platform for the exchange of teaching ideas.
... highlights the importance of science and technology in schools and among the public.

The main supporter of Science on Stage is the Federation of German Employers' Associations in the Metal and Electrical Engineering Industries (GESAMTMETALL) with its initiative think ING.

Join in - find your country on
WWW.SCIENCE-ON-STAGE.EU
www.facebook.com/scienceonstageeurope
www.twitter.com/ScienceOnStage

Subscribe for our newsletter:
www.science-on-stage.eu/newsletter

SCIENCE ON STAGE GERMANY

MAIN SUPPORTER OF SCIENCE ON STAGE GERMANY
think ING.
Die Initiative für Ingenieurnachwuchs

Proudly supported by
SAP®