STEPHEN KIMBROUGH · DAMJAN ŠTRUS

SHOOT TO THRILL

penalty shoot-out, combinatory, game theory
mathematics, computer science, physics
14–18 years

1 | SUMMARY

This project requires students to calculate the probability of a successful penalty shot, taking into account all of the internal and external influences (i.e. geometry, reaction time, choice of side).

The students must also find the perfect line-up for a penalty shoot-out as well as a “fair” alternative to it.

2 | CONCEPTUAL INTRODUCTION

The penalty shoot-out was introduced into the FIFA World Cup football rules in the 1970s.

It applies if a game is tied after extra time, i.e. the extra period of play added on to a game when the score is level after regular time. Before the introduction of this new rule, the winner was decided by the toss of a coin.

Penalty shoot-outs are among the most thrilling situations that can occur during a football match.

In this unit we will analyse how to maximise the outcome for a specific team.

The unit is divided into two parts. In the first part, the students calculate the probability of scoring a goal with a single shot. In the second part, they learn how the penalty shoot-out can be optimised.

3 | WHAT THE STUDENTS DO

3 | 1 Single penalty

To find out how high the probability of scoring is, we need to divide the penalty shot into two independent motions, those of the goalkeeper and those of the penalty taker.

First we assign probabilities to the goalkeeper on the basis of trigonometry.

The football goal is a rectangle with a width of 7.32 m and a height of 2.44 m. The height of the average goalkeeper is about 2 m and he has an arm span of about 2 m. The students can then compare the area covered by the goalkeeper with the area of the football goal. This yields the probability of the goalkeeper preventing a goal.

A second aspect is the goalkeeper’s reaction time and how long it takes him to reach the ball.

Students should begin by guessing where the best spots to aim the shot would be. The answer is: the upper corners of the goal. Then they have to use trigonometry to calculate the distance to that point. The time the ball travels can be calculated ($t = \frac{s}{v}$), with the assumption that the average velocity of the ball is 100 km/h.

The goalkeeper has that amount of time to react and jump into the right corner.

The students measure their own reaction time with a ruler that is dropped by one student and caught by a second student (see p. 30). Using the distance the ruler has travelled, the reaction time can be calculated as

$t = \sqrt{\frac{2h}{g}}$.
g: gravitational acceleration; $g = 9.81 \frac{\text{m}}{\text{s}^2}$
t: time [s]
h: distance covered [m]

Subtracting this reaction time, the goalkeeper has the remaining time to cover the distance to the ball. The latter has already been calculated, such that he or she has to have an initial velocity of $v = \frac{x}{t}$ in order to reach the ball. An athlete’s average speed when jumping is approximately 16 km/h.

By comparing the two velocities, the students can see that the goalkeeper would never be able to reach the ball. This yields the conclusion that the goalkeeper cannot allow for any reaction time and must choose which corner to dive toward before the penalty has been taken.

Students divide the goal into two halves and calculate the probability of preventing the ball from going in one half of the goal, using the same method as above. This can also be calculated again, after dividing the goal into thirds.

It is hard for the penalty taker to estimate probabilities, but in general it can be said that a left-footed penalty taker will aim better at the right corner, and a right-footed penalty taker at the left corner.

The students can accumulate data by shooting 10, 20 or more times at an empty goal and calculate the accuracy of their shots.

The students should then write a program, or use the source code that can be found in the appendix [1], to simulate a penalty shot. The students first have to enter their probability figures. For both the goalkeeper and the penalty taker, the direction of the shot is altered by randomness. Bearing in mind the law of large numbers, the probability of scoring a goal in a penalty shoot-out can be determined by increasing the number of shots. On this basis, the students can explore the question of whether altering the strategies for shooting will lead to a higher or a lower

FIG. 1 Perspective of the penalty taker

FIG. 2 Perspective of the goalkeeper

accuracy. The students can compete against each other with their respective codes.

3 | 2 Penalty shoot-out

Penalty shoot-outs always take the same form. Five players from each team are nominated to take penalties in a fixed order. A coin is tossed to decide which team gets to choose which team should shoot first. The teams then take turns shooting a penalty.

FIG. 3 Sequence of a penalty shot

The students are given a list of players with their average scoring probabilities. They choose five of these players and determine the line-up in which they will shoot. Two of the students compete against each other in a game that has been programmed in Scratch 2 [2]. Afterwards the students will have to prove that their line-up is the best possible one. Since the mean probability for scoring a goal is

$p = \frac{(p_1 + p_2 + p_3 + p_4 + p_5)}{5}$ all of the line-ups are equal.

The problem in real-life football compared to computer simulations is that the pressure on each penalty taker rises as the penalty shoot-out progresses. This value can be set at about 5 %. This will lead to the following equation for the mean probability:

$$p = \frac{(p_1 + 0.95p_2 + 0.90p_3 + 0.85p_4 + 0.80p_5)}{5}.$$

Since we have $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$ possible line-ups, the students must figure out a way to optimise the result. It should be up to the students to find a solution to the problem, although having the weakest penalty taker first and ascending to the strongest last is in fact the best solution.

With this in mind, the students can alter the Scratch 2 program to adapt it to their needs.[2]

The next variable that plays a role here is the psychological effect if the team that shoots first scores a goal. This situation puts even more pressure on the next penalty taker.

Next, the students can compare two teams of equal strength, changing the program and simulating it many times. This yields the conclusion that the team that begins has a higher chance of winning the penalty shoot-out.

The students should finally have a debate to determine a fair rule for a penalty shoot-out. They should test the rule with the program mentioned above and find out whether five shots are enough to reach a satisfactory outcome.

The fairest sequence for Teams A and B, each one with eight players, would be AB BA BA AB. This is also known as the Thue-Morse sequence. The sequence of the teams shooting has to be altered, and the alteration itself also has to be altered.

4 | CONCLUSION

The students will learn how to model a real-world scenario and to analyse it mathematically. They will also learn how to use their programming skills to solve problems generated by complex situations and to write their own simulation of a penalty shoot-out.

5 | COOPERATION OPTIONS

Students can organise a competition in class or against another school to see which penalty shoot-out strategy is the best (see *3.1*).

Another idea could involve an attempt by the students to "improve" the rules of football by changing the size and shape of the goal. What would happen to the penalty shoot-out if the goal were round or triangular?

REFERENCES

[1] www.science-on-stage.de/iStage3_materials
[2] https://scratch.mit.edu/scratch2download/

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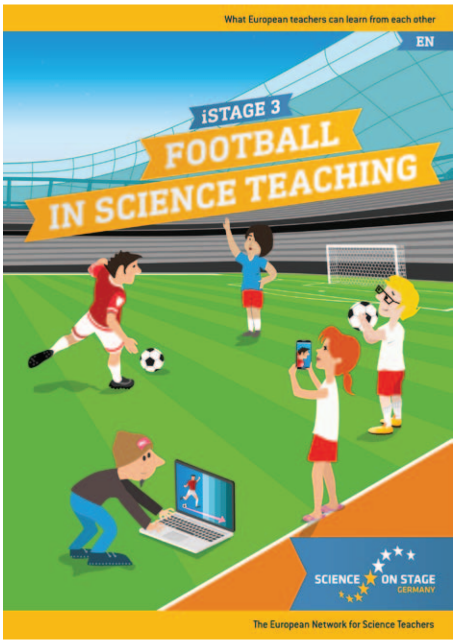

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