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SCREWBALL PHYSICS

Magnus effect, fluid dynamics
physics, mathematics
16–19 years

1 | SUMMARY

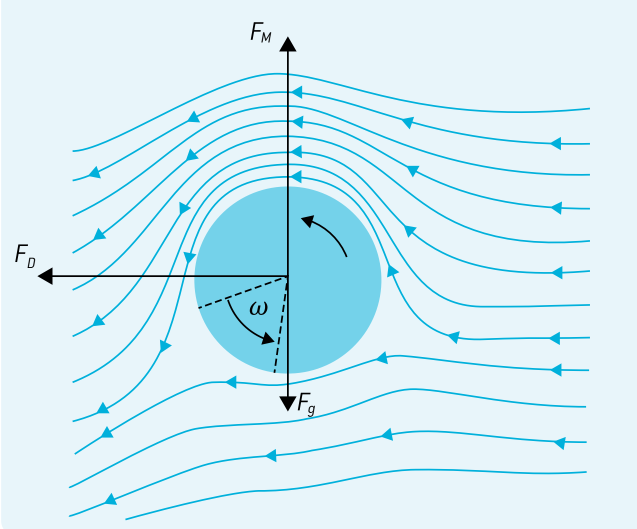
A rotating ball moving through the air will curl due to the Magnus effect, a force acting perpendicular to the direction and the rotational axis of the ball. Here we present some practical experiments, simulations and methods to calculate the trajectory.

2 | CONCEPTUAL INTRODUCTION

In June 1997, Roberto Carlos scored an infamous goal in a 35 m free kick that still baffles the viewer.[1] How can the ball behave like this, going off in one direction and then magically curling toward the goal? The answer is that the ball is spinning in the air and is subjected to the Magnus force. If you would like to see a free introduction to free kicks from Master Roberto himself, we highly recommend his video from the UEFA Training Ground home page.[2] If you want a free introduction to the Magnus force, continue reading.

To analyse the trajectory of a ball, we need to assess three forces acting on the ball: gravity F_g, the Magnus force F_M and the drag force F_D.

FIG. 1 Forces [3]



The gravitational force is simply given by Newton's second law, $F_g = mg$, where m is the mass of the ball and g is the gravitational acceleration.

The Magnus force F_M occurs due to differences in pressure on opposing sides of the ball. The changes in pressure can be described via the Bernoulli principle. For a point on a surface moving through a medium with the velocity v, the total pressure p is equal to the surrounding static pressure p_0 plus the dynamic pressure q (**EQ 1**), where ρ is the density of the medium, in our case the density of air. But when a ball or cylinder with a radius R is rotating (with an angular velocity of ω in radians per second), a point on the surface on one side of the ball is subjected to a higher flow of air ($v + \omega R$) than the opposing point on the other side ($v - \omega R$). Hence we can derive the difference in pressure $\Delta p = 2\rho\omega vR$ from **EQ 1**.

$$p = q + p_0 = \frac{\rho v^2}{2} + p_0 \qquad \textbf{(EQ 1)}$$

$$\Delta p = \left(\frac{\rho {v_2}^2}{2} + p_0\right) - \left(\frac{\rho {v_1}^2}{2} + p_0\right)$$

$$= \frac{\rho\left((v+\omega R)^2 - (v-\omega R)^2\right)}{2} = 2\,\rho\omega vR$$

$$F_M = \Delta pA = (2\rho\omega vR)A$$

For a cylinder: $F_M = 4\rho\omega vR^2h$. **(EQ 2)**

For a sphere: $F_M = 2\rho\omega v\pi R^3$. **(EQ 3)**

The pressure acting on the surface will constitute F_M. Without going too deeply into the maths behind it, we only need to address the forces acting perpendicular to the fluid flow. Any force acting in a direction other than perpendicular to the flow will be cancelled out by another opposing force due to symmetry. Hence, we only look at the effective cross area A of the object. For a ball, A will simply be a circle with the radius R (used in **EQ 3**); for a cylinder, A will be a rectangle with the height $2R$ and width h (used in **EQ 2**). In terms of vectors, $\vec{F}_M$ is proportional to the cross product of the directional velocity and the angular velocity.

Finally, we have to assess the drag force F_D. Drag is complicated, as the airflow can be laminar or turbulent, depending to a large extent on the shape of the object and the nature of the fluid it is moving in. For our experiments it suffices to assume that the flow is laminar (as in **FIG. 1**) and use the standard drag equation where the force is directed in the opposite direction to v and proportional to the velocity: $F_D = \beta v$. β is a constant that depends on the properties of the fluid and the dimensions of the object; for a football in air it is $\beta = 0.142 \frac{\text{kg}}{\text{s}}$ [4].

3 | WHAT THE STUDENTS DO

Here we present three different options to demonstrate the Magnus effect. All of these experiments can be carried out as simple demonstrations, but you can also record the experiments and use our models to analyse the trajectories. In that case, make sure you record with a stationary camera at the same height as the objects and perpendicular to the trajectory, and at least a few metres away, in order to minimise angular distortion. The movie can then be analysed with a motion tracking program. We recommend Tracker [5]. You can find detailed instructions on how to use Tracker in our first iStage book [6]. There is an excellent app called VidAnalysis [7] that will record the trajectory and carry out the analysis directly on an Android device (**FIG. 2C**). The data can also be exported for further analysis; here we use the free software GeoGebra [8].

FIG 2 Slope cylinder

3 | 1 Cylinder experiments

Make different cylinders by using A4 or A3 sheets of paper and glue. Mount a tilted board and let the cylinders roll down the slope to obtain a free fall with rotation **(FIG. 2A)**.

The students can examine what happens if they change the inclination of the slope, the radius or the height of the cylinder. The students can experimentally determine the parameters that will visibly give a greater effect and correlate this to **EQ 2**, or they can go further, extract the data, and proceed with data analysis (Model II) as described later on.

The Magnus effect in water **(FIG. 3)** is even more impressive because of the higher density of the medium. The cylinder must have a higher density than water, and a rugged surface to increase the friction. We used a solid Teflon rod with Velcro glued to the surface. To adjust the weight of the cylinder you can glue coins to the ends of the cylinder.

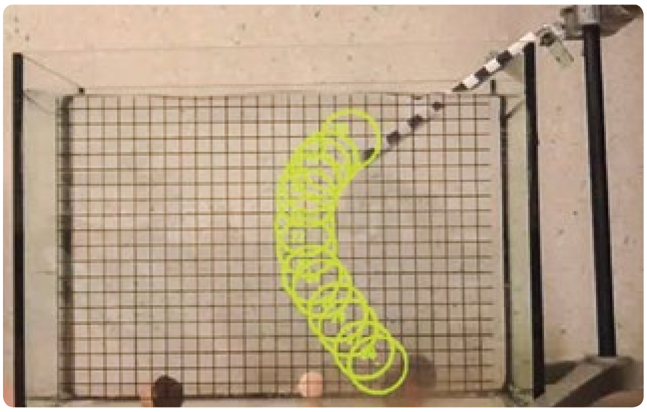


FIG. 3 The Magnus effect in water

An even more spectacular but trickier setup is to glue or tape the bases of two Styrofoam cups together so that you get a cylinder with a waist in the middle.[9] Coil a string around the waist and release the cylinder into the air by jerking the string (**FIG. 4**; there is also a link to a movie on our GeoGebra page [10]). This requires some practice, but the result is spectacular. The experiment is less reproducible compared to the other cylinder experiments, as the trajectory depends on the angle and how hard you jerk the string. Nevertheless, you can analyse the successful trajectories individually. In **FIG. 4** the flying cups go into a circular movement. If the Magnus effect is substantially greater than the gravitational pull, F_M behaves like a centripetal force. This useful assumption will be used later during data analysis.

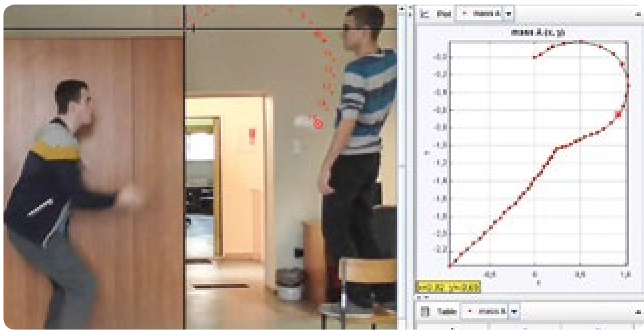


FIG. 4 Flying cups

3 | 2 Data analysis

We developed different mathematical models to analyse the trajectories. These models are directly accessible online from our iStage 3 GeoGebra page [10]. We strongly recommend that you open them before continuing to read this text. They will run directly in your browser; just click on the link.

In all the calculations, we have assumed that the rotation is constant during flight. We then make two simplified models based on different assumptions:

Model I: As in the question-mark trajectory with the flying paper cups **(FIG. 4)**, F_M will behave as a centripetal force, and the calculated trajectory of the object will be a circle with the radius r. This assumption is also reasonable in a penalty kick situation, where the total velocity of the ball stays roughly the same. Some of the energy is lost due to turbulence, so we need to introduce a constant C_s to describe this loss.

Then we have:

$$F_M = C_s 2\rho\omega vRA = \frac{mv^2}{r}.$$

For a sphere: $r = \dfrac{mv}{2C_s \pi \rho \omega R^3}$. **(EQ 4)**

For a cylinder: $r = \dfrac{mv}{4C_s \rho \omega h R^2}$. **(EQ 5)**

You can see the trace from **FIG. 4** in our GeoGebra model (flying cups) and change the centre of the circle and C_s. Play around with the parameters to get the best fit; the model will calculate r from **EQ 5**. For our data the best fit is $C_s = 0.86$.

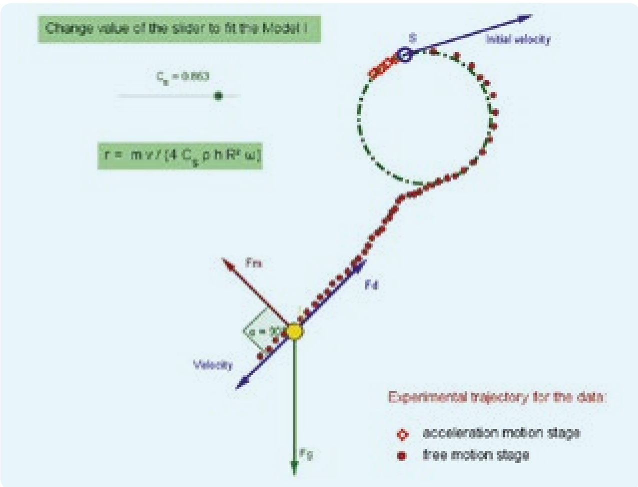


FIG. 5 Flying cups analysis

Model II: To simplify the calculations for the experiment with the paper cylinder **(FIG. 2)**, the students can assume that the Magnus effect is pulling mainly perpendicular to the initial direction of motion, and that the cylinders have reached maximum velocity when they fall. With these assumptions, F_D and F_g cancel out, and the Magnus effect can be regarded as acceleration a in the y-direction, so the calculated trajectory will be a parabolic curve:

$$y = \frac{a}{2v^2}x^2 \implies y = C_s \frac{\rho \omega R A}{mv}x^2.$$

For a sphere: $y = C_s \dfrac{\pi \rho \omega R^3}{mv}x^2$. **(EQ 6)**

For a cylinder: $y = C_s \dfrac{2\rho \omega h R^2}{mv}x^2$. **(EQ 7)**

This is a simplification, but it will give us a similar C_s as in our other model.

On our GeoGebra page **(FIG. 6)**, we have staged a recreation of Roberto Carlos' infamous free kick. You can play around with almost all the parameters to change the setup (distance, angle, the size of the goal, C_s, speed, rotation, position of the four-man wall, etc.). The analysis will show the calculated trajectory of both Models I and II, this time using **EQ 4** and **EQ 6** because we are looking at a ball instead of a cylinder. Challenge your students to find the best values for a given setup, or ask them to find the conditions where the models give different trajectories and ask them to explain why. (You will find out that the models differ when you give the ball very low velocity and high rotation).

3|3 Simulations

2D simulation: After some hands-on experiments the students can simulate the Magnus effect. Download the Java program [11]. In this simulation the students can modify the initial velocity, the angle, the drag coefficient and the angular frequency. The rotational direction and the forces acting on the ball are as shown in **FIG. 1**. In **FIG. 7** we show three examples of the trajectories at 30° with a frequency of 0, then 5 and 10 $\frac{\text{rev}}{\text{s}}$. You can

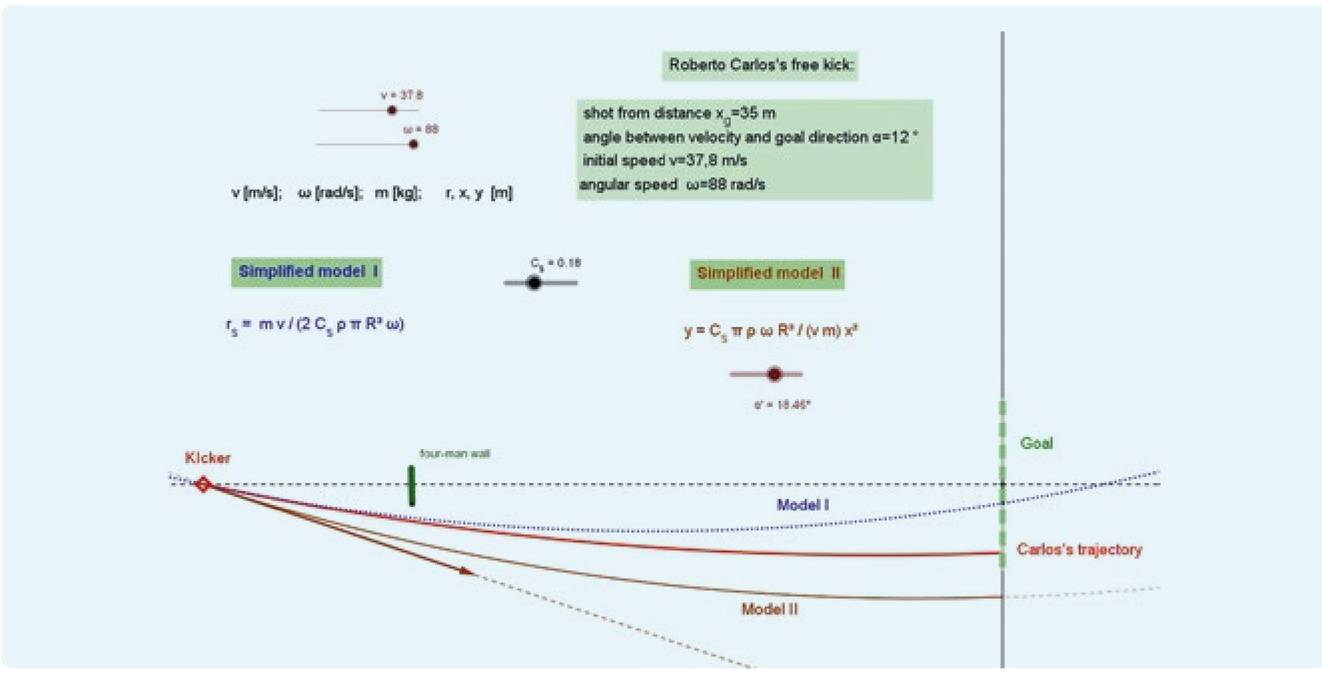


FIG. 6 Free kick analysis

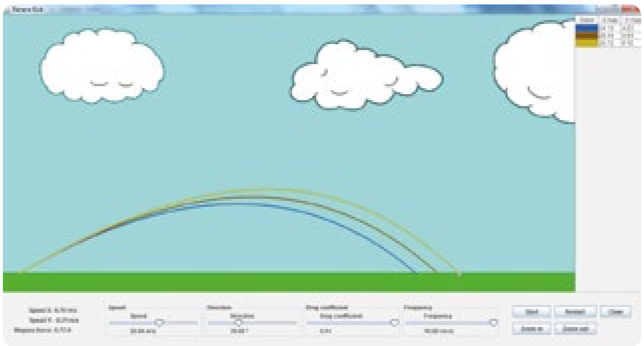


FIG. 7 2D simulation

see that the values of x_{max} and y_{max} are increasing if the frequency is increasing too.

3D simulation: Once again we have recreated the trajectory of Roberto Carlos' free kick **(FIG. 8)**. Now you can try it out yourself by downloading the respective Java program [11]. Later, you can try a different version [11] without the kick, but you can change the parameters freely to see what influence they will have on the trajectory.

In 3D, things rapidly become more complex. In the two-dimensional model the ball can only have top spin or under spin, so the trajectory and the Magnus force will always act in the same plane. In the three-dimensional model the Magnus effect will curve the trajectory of the ball, but the angular momentum of the spin will always be conserved, as the ball behaves like a gyroscope. So the angle between v and ω will be different at different points of the trajectory, which will give the ball a more complex trajectory. Unlike the GeoGebra calculations, this program simply calculates all the forces numerically in each frame on the basis of the values in the previous frame. The program is written in Processing [12], a simplified version of Java.

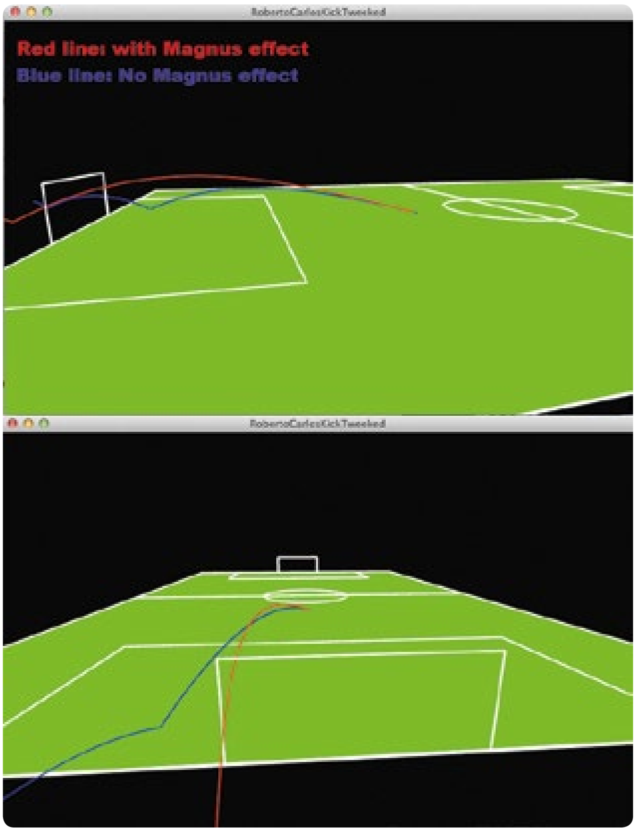


FIG. 8 3D simulation

4 | CONCLUSION

On the football field the trajectory of a football is complex and depends on a whole array of factors. In order to study this in the classroom the students have to break it down into manageable components using models and simplifications. These experiments, models and simulations give an insight into what we can conclude from working with a scientific method: If we assume that the game is played under water or that the football can be replaced by two paper cups we come very close to the explanation how Roberto Carlo manages to curve the ball.

5 | COOPERATION OPTIONS

On our iStage 3 GeoGebra platform [10] you can find information about how to obtain a copy of our GeoGebra files and how to use them. We propose a challenge: obtain the highest Magnus effect possible for the flying paper cups experiment. That corresponds to finding the highest value for C_s, as close to 1 as possible. You can share your analysis, results and models [11].

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